Performance Robustness of Manipulator Collision Controller

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Abstract

In this paper, we propose that the manipulator impact control problem be approached from a stochastic optimal control perspective. The reason is that not only is such approach be able to model uncertainties in contact environment, force sensing, as well as manipulator dynamics, the controllers obtained is optimally robust in terms of performance. This result is verified by analyses and simulations.

1. Introduction

Today, as robot manipulators are expected to interact more with the environment for partially constrained tasks, the necessity of high performance collision controllers becomes more and more significant. For manipulator *collision* control, it is desirable to have a controller which can make contact fast without bouncing despite the uncertainty in the location of collision surface. Furthermore, the transient impact force and the steady state force error should be minimized despite. the uncertainties in the environment dynamics as well as force sensor delays. In this paper, wc show that the controller derived based on stochastic optimal control approach is optimally robust in terms of performance for a given uncertainties. Performance robustness implies here as how well the system can maintain good performance under the presence of uncertainties. Previous work related to manipulator impact as well as force control problem mainly focus on guaranteeing stability in the. presence of uncertainties. The performance robustness issue is very often ignored even though it is also an important issue. in the following, to illustrate that stochastic optimal approach results in a controller which is optimally robust in terms of performance, a simple example problem taken from [1] will first be re-examined in detailed, In [1], a cost functional of the force error is taken over the period when contact has been established and the optimal approach velocity is derived for a given environment and controller design. Extending the problem in [1], wc allow the. controller gains to vary and the environment dynamics and approach velocity (due to environment location uncertainty and force sensor delay) to be uncertain. We show that by minimizing the expectation of the cost functional, a controller which is optimally robust in terms of performance to the uncertainties in approach velocity and environment dynamics can be derived. Since the approach velocity depends on the control policy used in non-contact regime as well as the environment location uncertainty and force sensor delay, its statistics in terms of pdf(probability density function) cannot be known unless we specify the control policy used in non-contact regime and include the collision surface location uncertainty and force sensor delay in our model. Notice also that the result of [1] is invalid if bouncing occurs. Therefore, this lead us to include the non-contact regime in the dynamic model which is just what [2] have been able to do. in [2,], the cost functional of the states of both contact and non-contact regime are optimized stochastically and thus the optimal approach velocity is obtained implicitly without having to evaluate its pdf.

The organization of the paper is as follow. in section 2, wc review previous works relate.ci to manipulator collision control. In section 3, we describe new results which arc obtained by extending [1] using stochastic approach. In section 4, a more general stochastic approach as described in [2] is discussed. in section 5, we show via simulations, that the controller obtained in [2] is optimally robust in terms of performance against collision surface location uncertainty and force sensor delay. Section 6 is the conclusion.

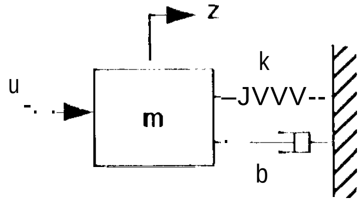
2. Previous Work

In the. past, many researchers have designed various form as well as impact controllers which guarantee stability in the presence of uncertainties but few address the performance robustness issue which is also important if the controller is to be implemented. Explicit force control (e.g. [3], [4]) which uses force sensor had been augmented in one way or the other to impedance controller [5] to achieve both accurate force tracking and good transient response when the environment dynamics are uncertain. Many performance and stability analyses assuming that the manipulator is attached to the environment were done [6], [7]. In this approach, it was expected that a precise knowledge of the environment is not needed because contact force. is directly controlled in a closed-loop fashion. However, this approach suffers from the following problems. [7] points out that at high gains, the impact will exhibit instability duc to non-collocation of actuator and sensor, unmodelled high order dynamics in both the environment and manipulator, actuator dynamics, and the discontinuity of the dynamics before and after impact. To deal with the stability and uncertainty problem, sliding mode controller [8] is used to achieve appropriate position control before contact and good force tracking after contact with only the knowledge of the upper bounds of the environment dynamic uncertainties. Similarly, [9] requires the knowledge of the bounds of the environment uncertainties and design a non-linear feedback based algorithm combined with explicit force control during different phases. [10] utilizing generalized dynamical system (GDS) theory developed an asymptotically stable discontinuous controller. Similar to the other methods, the performance of the controller also depends on the bounds of the environment uncertainties. [11] developed an adaptive nonlinear controller within the framework of GDS. They utilize a collision model as a feedforward signal to reduce the impact forces during collision and a model-based adaptive controller to realize good performance during all phases of the contact tasks. However, since all of these control schemes arc non-linear ant]/or discontinuous schemes, oniy stability arc proven and the performance is hard to predict. In our opinion, stability is essential but equally important is that the relationship between the amount of knowledge about tile environment and the performance of any controller must be established systematically before the control scheme can be used reliably. In the following, wc shall establish the notion of optimality of performance robustness and how it can be ultilized via a simple example.

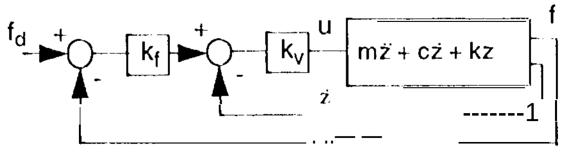
3. Stochastic Approach 1

In [1], it was observed that the performance of a linear controller in contact mode is directly related to the approach velocity which is just the velocity of the manipulator at the time of collision. This fact was observed some time ago [12]. in [1], by evaluating the. cost functional of the state trajectory of a simple force and velocity feedback system in contact mode analytically, the optimal approach velocity for a given controller design can be obtained anti found to be proportional to the force. command. However, [1] did not dealt with tile question of how to generate the optimal approach velocity when there is uncertainty in the collision surface location and force sensor delay. Moreover, the optimality of the approach velocity is valid only for a particular controller and assuming tile environment dynamics arc known a priori. How do wc design the controller that is able to maintain good performance (i.e. good force tracking and transient suppression) in the presence of a given uncertainties in the environment dynamics, collision surface location and possibly force sensor delay? To answer this question, wc must first be able to predict the performances in terms of the uncertainties involve. Therefore, wc extend the problem in [1] to include uncertainties in environment dynamics and approach velocity and allow the controller gains to vary. Then, wc minimize tile cost functional both deterministically and stochastically and compare the results in terms of performance anti sensitivity to parameter variations.

Figure 1 shows a simple model of the manipulator and the environment during contact along with the closed-loop block diagram on the right according to [1]. The manipula-



Simple physical model of manipulator & environment



Closed-loop block diagram

Figure 1 Simple manipulator and closed loop control model

tor is assumed to be a rigid body while the environment is just a spring-mass-damper system. The state space description of the error dynamics of the block diagram in Figure 1 is

$$\dot{\tilde{z}} = A\tilde{z} \quad (1)$$

$$y = C\tilde{z} \quad (2)$$

where $\tilde{z} = z - z_{eq}$ (3)

z_{eq} is the equilibrium position induced by the constant force command f_d and y is the force error, $f - f_d$. A is the closed-loop error dynamics of Figure 1. To reflect the performance in terms of the force transient, [1] defines a quadratic cost functional in terms of the force error:

$$J = \tilde{z}_0'\left[\int_0^\infty \left(e^{At}\right)' C'C\left(e^{At}\right)dt\right]\tilde{z}_0 \quad (4)$$

where

$$\tilde{z}_0 = \left[-\frac{k_v k_f}{k\left(1 + k_v k_f\right)} f_d \qquad v_0 \right] \qquad (5)$$

and v_0 is the approach velocity, i.e. $\dot{z}(0)$. In this framework, $t = 0$ corresponds to the time of collision. According to (4), J is a function of the system parameters, m, c, k, V., and the control gains k_v and k_f. For a given set of system parameters, m, c, k and control gains, [1] obtain the optimal v_0 by minimizing J w.r.t. v_0. In faCt, if we allow the controller gains to vary, there. will be optimal control gains k," and k_f^* as well as optimal approach velocity V. by minimizing **J** w.r.t. k_v, k_f, and V. simultaneously.

If there were uncertainties in the collision surface location and force sensor delay, the approach velocity obtained in such way cannot be implemented because it becomes a random variable whose statistics (e.g. pdf) depends on the collision surface location uncertainty, force sensor delay and the control policy used in non-contact regime. For illustration purpose, we shall assume in this section that somehow the pelf of V. had been obtained. Suppose there are uncertainties in the environment dynamic parameters m, c, k with their pdf's known as well, J in (4) becomes a function of several random variable.s, m, c, k, V. and deterministic variables k_v, k_f. As a result, to optimize the cost w.r.t. all these variables, some measure of the random functional J is needed. We define E { J }, the expectation of J, as the cost functional and obtain optimal control gains k_v^{**} and k_f^{**} by minimizing E { J } w.r.t. k_v and k_f. So we have two sets of controller designs, one obtained deterministically and one stochastitally. In the following, we observe that the performance of controller k_v^{**} and k_f^{**} obtained stochastically by minimizing E { J } is less sensitive to variations in m, c, k and v_0 than k_v^* and k_f^* obtained deterministically by minimizing J assuming m, c, k, V. equal to the corresponding mean (or nominal) values. Also, we observe that the. performance of k_v^{**} and k_f^{**} is close to that of k_v^* and k_f^* within the uncertainty region of m, c, k and V. around their nominal values.

Before we proceed, we need to consider the constraints namely actuator saturation and sensor noise. In (4), J is a function of m, c, k, V_o, k_v and k_p, where

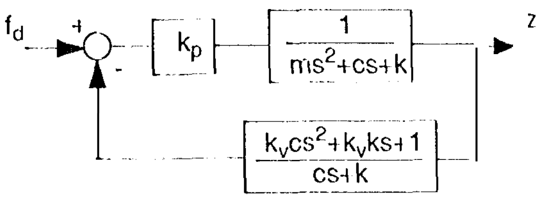


Figure 2. Rearranged block diagram of Fig.1

$k_p = k\ k_f$. we observed that if $k_p \to \infty$, $J \to 0$ which means that the force error converge to zero infinite.ly fast. This is impossible because of actuator limitation and noise in the sensor. In fact, if we rearrange the block diagram in Figure 1 by combining the two loops as shown in Figure **2,** it is obvious that magnitude of k_p must be limited to avoid large. loop gain which amplifies noise and cause actuator saturation .Therefore k_p can be considered as a constant equals to the upper bound and we can just vary k_v during optimization. Also, when k_p and k_v are positive, the system is passive and guaranteed to be stable and thus we only consider positive k_p and k_v from now on.

Coming back to the illustration of the claim made earlier, suppose the system parameters m, c, k, V. are uniformly distributed with lower and upper bounds $(\bar{m} - \delta m, \bar{m} + \delta m)$, $(\bar{c} - \delta c, \bar{c} + \delta c)$, $(\bar{k} - \delta k, \bar{k} + \delta k)$, $(\bar{v} - \delta v, \bar{v} + \delta v)$. The means are thus $\bar{m}, \bar{c}, \bar{k}$, V.. Assume arbitrarily that the value of m, c, k, v_0 be 1 and let $k_p = 1$. Plotting J vs. k_v, we obtain curve (a) in Figure 3. Then we let the system parameter be uncertain with means and standard deviations all equals 1 and 0.25 respectively. Plotting E { J } vs. k_v, we obtain curve (b) in Figure 3. From Figure 3, k_v^* and k_v^{**} are found to be around 2.25

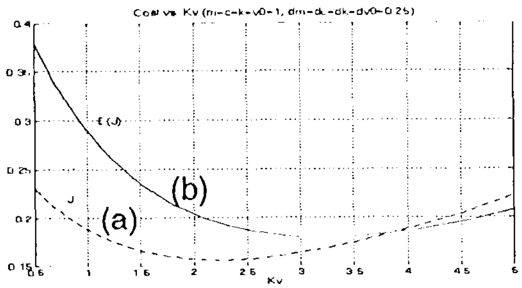


Figure 3. Plot of Cost against control gain k_v

and **3.25** respectively. Now substituting k_v^* and k_v^{**} back into J and allow the system parameters to vary, the result will be two surfaces in the system parameter hype.rspace. For illustration, we show the surfaces in 3-D plots as shown in Figure 4a-c. In each plot, we only plot the cost against two parameters, i.e. c & v_0, m & v_0, and k & V_o. In general, if we increase k_v, the surfaces such as those shown in Figure 4a-c will be flatter (less variation in J) but the overall height from zero increases indicating that the controller becomes more robust to system parameter variation but worse in performance. Thus, k_v^{**} is more robust to system parameter variation but worse in performance than k_v^*. This is illustrated in Figure 4. Notice that the variations of J associated with k_v^{**} are smaller than that of k_v^* which indicates that the controller k_v^{**} is more robust in terms of performance than controller k_v^*. Even though the performance of k_v^{**} is somewhat worse than that of k_v^*, the performance. reflected by the height of the cost associated with k_v^{**} within the uncertain region (i.e. 1 ± 0.25) around the nominal value of the parameters is close to that of the k_v^*. This shows that by optimizing the expectation of the cost, one is really obtaining both the

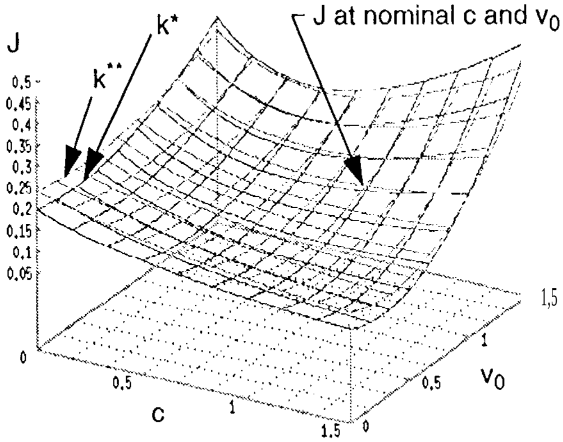


Figure 4a. Comparing J vs. V. and c for k* and k**

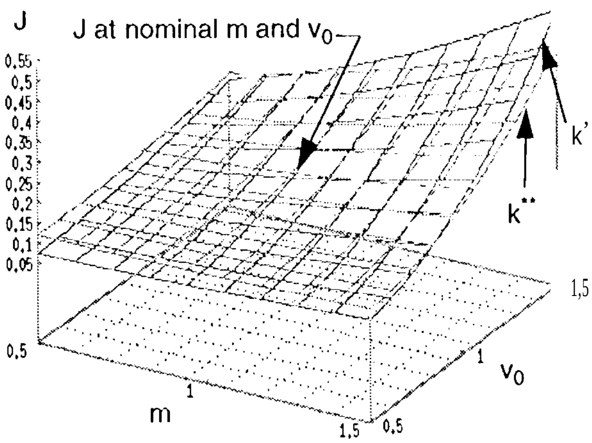


Figure 4b. Comparing J vs. V. and m for k* and k**

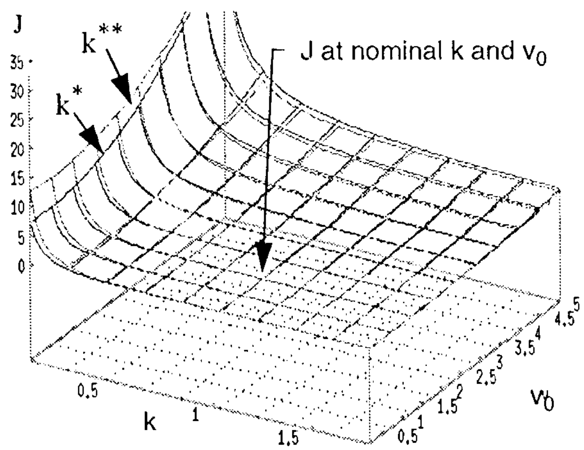


Figure 4c. Comparing J vs. V. and k for k* and k**

optimal performance and robustness according to the probability distribution of the uncertainties. Notice that if wc only minimize the sensitivity deterministically by increasing k_v continuously, it only leads to a large loop gain which resulted with a flat surface in J but the cost will be infinitely high. When minimizing the expectation of the cost, wc will not overdo the sensitivity minimization but only doing it optimally according to the pdf of the uncertainties involved while maintaining good performance. 'l'his simple example illustrates the. notion that stochastic optimal control approach (o the manipulator impact control problem yields a controller which is optimally robust in terms of performance for a given uncertainty. However, the controller could not be obtain in this way because the pdf of the approach velocity cannot be obtained without specifying the control policy before contact, the collision surface location uncertainty and force sensor delay. Also, if bouncing occurs, the result obtained so far becomes invalid. Therefore, we need to model the system dynamics more generally so that both contact and non-contact dynamics as well as the collision surface location uncertainty and force sensor delay can be included. in such way, there is no need to evaluate the pdf of the approach veloci(y and the optimal velocity can be obtained implicitly when the optimal control policy in non-contact regime is obtained. In [2], wc derived the so called "Jump Impact Controller" by minimizing the expectation of the quadratic cost functional of just such model which contains the non-contact and contact dynamics as well as the collision surface location uncertainty and force sensor delay. Therefore, according to our observations in this section, such design should be optimally robust in terms of perfor - mance. In the following, wc first briefly describe the Jump impact controller of [2] with some slight corrections and then investigate its performance robustness via simulations in section 5.

4. Stochastic Approach 11

According to [2], our system dynamics is described by tbc following state space equations,

Plant: $\dot{x} = A_1 x + Bu$, when $a \le 0$

$= A_2 x + Bu + B_f d_0$, when $\alpha > 0$

Observation: $y = Hx$

Regime indicator: $\alpha = Cx + \eta$

where $x \in R^7$, $y \in R^3$, $u \in R^1$. C and H are 1×7 and 3 X 7 matrices and thus the regime indicator α is a scalar while η which depends on the collision surface location, d_0, is a random variable. The matrices A_i, 11 and vectors C, B_i arc constant in time and the pairs (A_i, B) and (A_i, H) arc stabilizable and detectable respectively. Since wc do not have complete state measurement and exact rep, ime indicator duc to force sensor delay, wc need to use observers to reconstruct the states and some regime detection logic using the force sensor measurement. Suppose we design for our system an observer in the following form:

$$\dot{\hat{x}} = A_1\hat{x} + Bu + L_1 H(x - \hat{x}) + \Gamma f_d \qquad \text{when } \theta \le \beta$$

$$= A_2\hat{x} + Bu + L_2 H(x - \hat{x}) + \Gamma f_d \qquad \text{when } \theta > \beta$$

where L_1 and L_2 are stable observer gain matrices. θ is the filtered force sensor measurement and β is the threshold of the collision detection logic. f_d is a constant which represents the desired force. Define a new augmented states $z = [\hat{x}\ \tilde{x}]'$ where $\tilde{x} = x - \hat{x}$ and augment the plant and observer dynamics, then we obtain:

$$\dot{z} = F(r)z + Gu + g(r) \qquad (6)$$

where $r =$ 1, when $\alpha \le 0 \cap \theta \le \beta$
2, when $\alpha > 0 \cap \theta \le \beta$
3, when $\alpha > 0 \cap \theta > \beta$
4, when $\alpha \le 0 \cap \theta > \beta$

and $\alpha = [C\ C] = C_z z$. Define an estimated regime indicator, $\hat{\alpha}$, which is based on the states of the observer:

$$\hat{\alpha} = C\hat{x} = \hat{C}_z z$$

Since. x and $\hat{x}$ are connected by the observer gains L_1 and L_2, there exist some regime transition model that relates *a*

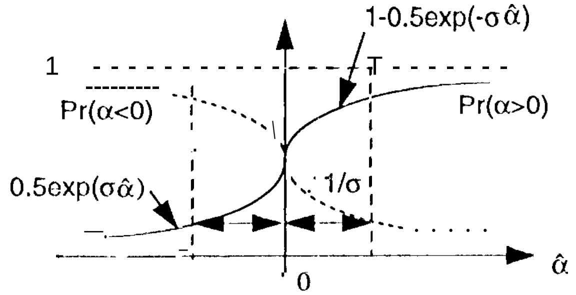


Figure 5. Probability curve of P(a)

and &. By making the assumption that the probability transition model depends on $\hat{\alpha}$ as shown in Figure S, the probability that r jumps:

$$\text{Prob}(r(\hat{\alpha}(t) + \Delta\hat{\alpha}) = j \mid r(\hat{\alpha}(t)) = i, z(t)) \qquad (7)$$

can thus be found. Intuitively speaking, this means that when the estimated states converge to the true states, the greater the value of &, the more probable *a is* greater than zero. To meet the requirements of manipulator impact control problem, we define the problem as to finding an admissible control that minimizes the expected value of a linear quadratic loss function,

$$J = E\left\{\int_{t_0}^{t_f} (z'Qz + u'Ru)\,dt \mid z_0, t_0, \hat{r}_0\right\}$$

subjected to (6), (7) where $\hat{r} = (\hat{\alpha}, O)$. Using stochastic maximum principle, we obtained an infinite-time suboptimal solution:

$u^* =$

$$-\frac{1}{2}R^{-1}B'\left[\left(P_1\hat{x} + b_1\right)\phi_b + V_a\phi_a\right] \text{ when } \hat{\alpha} \le 0 \cap \theta \le \beta$$

$$-\frac{1}{2}R^{-1}B'\left[\left(P_2\hat{x} + b_2\right)\phi_a + V_a\phi_b\right] \text{ when } \hat{\alpha} > 0 \cap \theta \le \beta$$

$$-\frac{1}{2}R^{-1}B'\left[\left(P_3\hat{x} + b_3\right)\phi_b + V_b\phi_a\right] \text{ when } \hat{\alpha} \le 0 \cap \theta > \beta$$

$$-\frac{1}{2}R^{-1}B'\left[\left(P_4\hat{x} + b_4\right)\phi_a + V_b\phi_b\right] \text{ when } \hat{\alpha} > 0 \cap \theta > \beta$$

where $\phi_a = P(\alpha<0)$, $\phi_b = P(\alpha>0)$

P_i satisfy some Riccatti equations, (for example)

$$P_1A_1 + A_1'P_1 - \frac{1}{2}P_1BR^{-1}B'P_1 + 2Q_1 + \sigma C'V_a'A_2 = O$$

b_i depends on P_i, the system parameters and f_d. V's satisfy the constraints:

$$A_1'V_a = 0$$
$$A_2'V_b = 0$$

5. Performance Robustness of Stochastic Approach 11

In this section, by examining the JIC design via simulations, we verify that JIC is optimally robust in terms of performance to the given statistics of the collision surface location uncertainty. Also, we show that there is a trade-off between performance and robustness when there are uncertainties in the dynamic model. As will be shown from the simulation results, the JIC can be designed to give excellent performance but very sensitive to uncertainties in dynamic modeling error, noise, sampling effect and collision surface location uncertainty. On the other hand, JIC can also be design to be quite robust to all these uncertainties but have to give up good performance in accord with the results of section 3. The parameters used in the simulation are obtained from the experimental data in [13]

We first investigate the robustness and performance

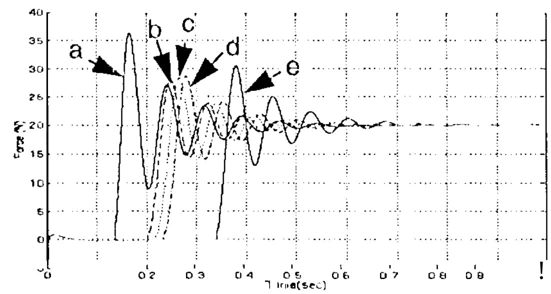


Figure 6. Performances when d_0 varies given σ = le-4

of JIC against collision surface location uncertainty. In the following simulations, we. assume that there is no modelling errors and d_0 is normally distributed with zero mean. First, wc let the 3-sigma value of $d_0 = 0.001$ m which makes $\sigma = 1$ c-4 (one can derive the relationship between the 3-sigma value of d_0 and design parameter σ which will not be shown here due to space limitation). This means that wc are very sure about the collision surface location (accurate to $\pm$ 1 mm). Then we vary do from -0.01 m to + 0.01 m as shown in Figure 6. From the results, wc can scc that the performance is pretty much the same when do is within ± 1 mm i.e. curves b, c, and d. Outside this range, the performance worsen as indicated in curve a and c. Next wc change the 3-sigma value of d_0 to 0.01 m which makes $\sigma = 1e\text{-}5$. Then we vary do from -0.03 m to + 0.03 m as shown in Figure 7. Again, from the results, wc

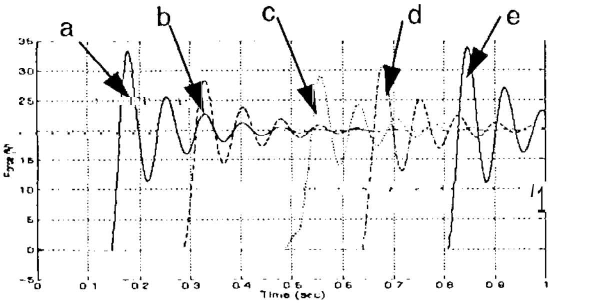


Figure 7 Performances when do varies given σ :

observe that when d_0 is within the range of the given pdf which corresponds to a certain design value of σ, then the performance is robust to the collision surface uncertainty i.e. curves b, c, and d. When do is outside this range, the performance deteriorate, i.e. curves a and c. This verifies that the JIC is optimally robust to collision surface location uncertainty for a give pdf of do.

If there is no uncertainties, the JIC can be designed to give good performances by increasing the magnitude of Q as shown in Figure 8. The magnitude of Q for curve a and

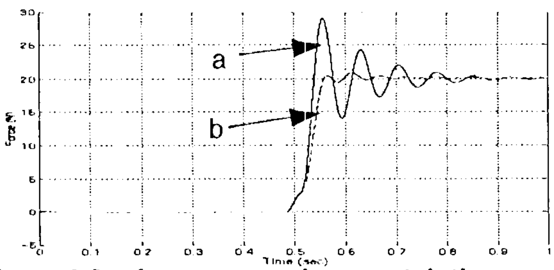


Figure 8 Performances w/o uncertainties w.r.t Q

b arc 1 and 1e5 respectively. It is clear that when the magnitude of Q is large, the performance is better. However, the trade off for good performance is poor robustness. In Figure **9** and 10, we compare the robustness of the two designs of Figure 8 in the presence of dynamic modelling errors and sampling effect.in Figure 9, when the magnitude of Q is large, and in the presence of uncertainties in the environment stiffness and collision surface, location (environment 4 times stiffer than expected and $d_0 = 0.005$ m), the manipulator keeps bouncing without being able to maintain contact, However, when Q is small, contact is maintain even though the performance is somewhat degraded. In Figure 10, the sampling frequency in the observer is 91 Hz instead of continuous. When the magnitude of Q is large, the performance is sensitive to sampling effect. When the magnitude of Q is small, the performance is more robust to sampling effects. From these simulation results, it clearly shows that the design with larger magnitude of Q is less robust to that of small Q similar to the effects of k_v in section 3. Finally, in Figure 11, wc show a

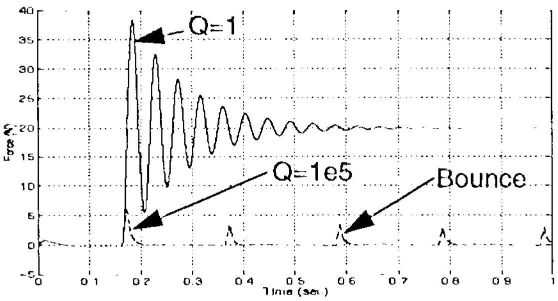


Figure 9 Uncertain environment stiffness (do = 0.005 m, k_w = 5e4 instead of 1.3e4)

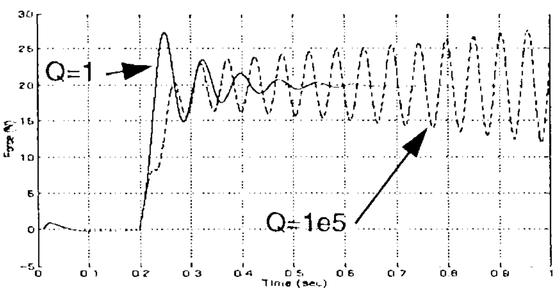


Figure 10 Sampling effect in observer (91 Hz, do= 0.001 m)

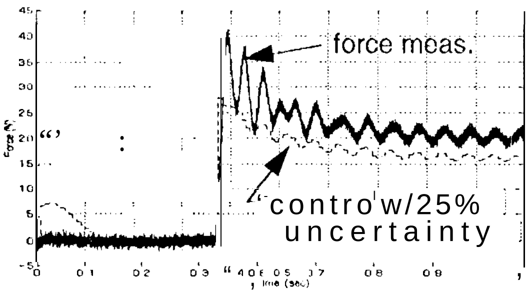


Figure 11. Performance w/ all kinds of uncertainties

simulation run with uncertainties in environment stiffness and collision surface location, sampling effects, input uncertainty, sensor noise and initial estimates error. We design the JIC in such a way that it gives good performance while robust to the uncertainties. Therefore, it is possible to adjust both σ and Q to obtain the require performance and robustness.

6. Conclusion

in this paper, we. dealt with the performance robustness issue for the manipulator impact control problem. We have shown that stochastic optimal control approach yields a controller optimally robust in terms of performance according to the statistics of the uncertainties. This idea is first illustrated using a simple model assuming contact mode. We observed by example that the controller obtained by minimizing the expectation of a cost functional of the force error is more robust in terms of performance against the variations of dynamic parameters and approach velocity than that obtained by minimizing the cost functional deterministically using the nominal value of system parameters and approach velocity. Even though the performance using the stochastic approach is somewhat degraded, it is about the same as that of the deterministic approach around the nominal value of the system parameters within the uncertainty range. However, this stochastic approach using just the contact mode dynamics does not yield an implementable control strategy because the pdf of the approach velocity depends on the control policy in non-contact mode as well as the collision surface location uncertainty and force sensor delay. Therefore, both non-contact and contact dynamics as well as collision surface location uncertainty and force sensor delay need to be included in the model before optimization is carried out. We show that [2] have done just that. As a result, using the approach of *[2], we* no longer need to evaluate the optimal approach velocity as it is implicitly implied in the optimal control policy in non-contact mode and an implementable controller derived from stochastic optimal contt 01 approach can be obtained. Through simulations, wc have shown that the controller is optimally robust in terms of performance against the collision surface location uncertainty and force sensor delay. Also, the performance and robustness of the controller against environment dynamic uncertainties can be adjust by changing the weight in the cost functional,

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